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Ambition through Challenge
Excellence through Curiosity

Calculation Policy for Mathematics

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The following calculation policy has been devised to meet requirements of the National Curriculum 2014 for the teaching and learning of mathematics, and is also designed to give pupils a consistent and smooth progression of learning in calculations across the school. Please note that early learning in number and calculation in Reception follows the “Development Matters‟ EYFS document, and this calculation policy is designed to build on progressively from the content and methods established in the Early Years Foundation Stage. The purpose of our Calculation Policy in setting out these aims is to ensure consistency in our approach to teaching calculation across the school. While compact methods may represent a significant achievement for some children it can present barriers for others. Each member of staff has a responsibility to ensure they give all children the opportunity to develop calculation methods that are appropriate to their age and ability.

Age stage expectations

The calculation policy is organised according to age stage expectations as set out in the National Curriculum 2014. At ********** this includes;

Stage 1 – Nursery and Reception
Stage 2 – Year 1 and 2
Stage 3 – Year 3 and 4
Stage 4 – Year 5 and 6

However, decisions about when to progress should always be based on the security of pupils' understanding and their readiness to broaden and deepen knowledge. Pupils who grasp concepts rapidly should be challenged through being offered rich and sophisticated problems before any acceleration through new content. Those who are not sufficiently fluent with earlier material should consolidate their understanding, including through additional practice, before moving on.

Providing a context for calculation:

It is important that any type of calculation is given a real life context or problem solving approach to help build children's understanding of the purpose of calculation, and to help them recognise when to use certain operations and methods when faced with problems. This must be a priority within calculation lessons.

Models & images and resources

At *********, staff must provide suitable resources for children to manipulate and explore how and why a calculation strategy works, and that helps them to describe and visualise or 'see' the method working. The equipment can include objects like counters, interlocking cubes, coins, counting sticks, bead strings, number lines, 100-squares, place-value cards, structural apparatus like base 10 blocks, Numicon, diagrams of shapes divided into fractional parts, and so on. An interactive whiteboard is also a powerful tool for manipulating images.

Selecting when and how to use and to withdraw resources and visual images is a key part of teaching. This involves planning how best to construct a blend of teaching approaches that are selected and designed to match intended learning outcomes and children's needs, and to take account of the context and organisation of children.

Visualisation is important in mathematics. The ability to visualise representations, pictures or images and then adapt or change them is an important tool for example when problem solving, pattern spotting and reasoning in mathematics.

Problem Solving - Heuristics

Providing pupils with opportunities to develop problem solving strategies is an essential part of teaching. By introducing the common heuristics to children, we will give them opportunities to 'see' and heuristic in operation and explore the strategy, helping them to become independent problem solvers.

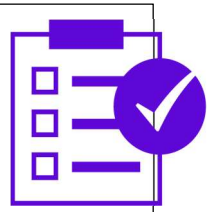
Heuristics

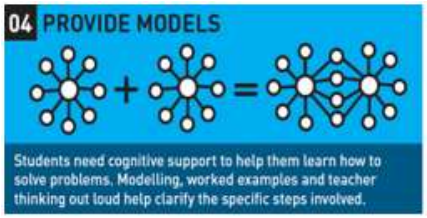
A set of problem-solving strategies that help us discover the best and most practical ways to solve problems

Annotation

Representation

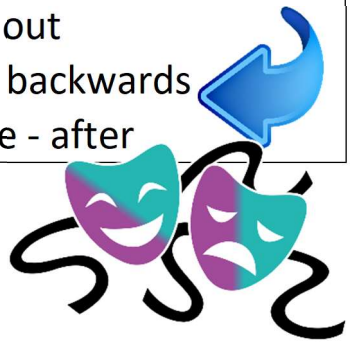
- Build it
- Draw a diagram
- Use a bar model
- Make a list
- Write a number sentence





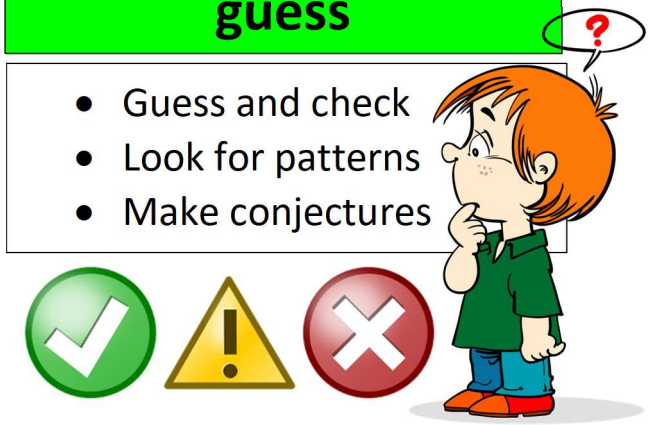
Go through the process

- Act it out
- Work backwards
- Before - after



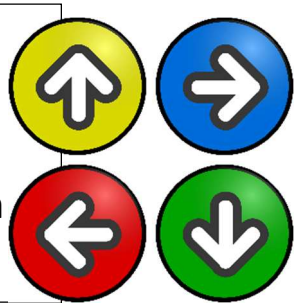
Make a calculated guess

- Guess and check
- Look for patterns
- Make conjectures



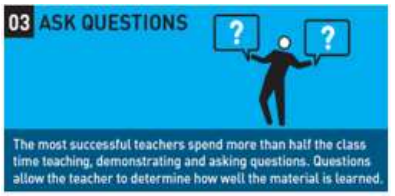
Change the problem

- Rewrite the problem in a different way
- Simplify the problem
- Solve part of the problem



Calculation decision making

Making decisions about strategies to be used needs to be modelled by the classroom teacher, talking through the internal questions they are asking themselves as they calculate the question is essential. By involving children in this reasoning approach, through questioning them, will assist children to ask themselves similar questions whilst they are working.



Can I do it in my head using a mental strategy?

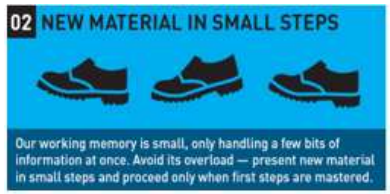
Could I use some equipment to help me?

Could I use some jottings or pictorial representations to help me?

Should I use a written method to work it out?

To work out a tricky calculation:		
Approximate	Calculate	Check it!

Progression in mental and written computation.

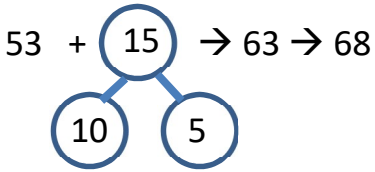


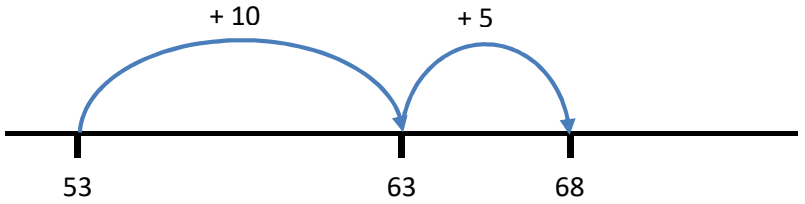
Children use informal strategies in familiar settings e.g. touching dots, moving beads, making finger patterns.
Begin to distance the setting from the child and pose problems involving 'bare numbers' for children to solve mentally.
Children learn to combine mental strategies with informal written jottings for more difficult tasks.
Using mental strategies and jottings we can develop semi-formal written strategies.
If appropriate for the learner, we can develop these strategies further to formal written computational algorithms.

Mental strategies for addition and subtraction (written and pictorial (jottings) included for clarification – as they are taught strategies initially)

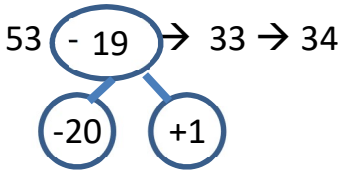
Using labels for different strategies supports discussion about thinking and draws attention to the number relationships used. As a school we have chosen to prioritise 5 key methods that will be modelled for adding and subtracting mentally two 2digit numbers.

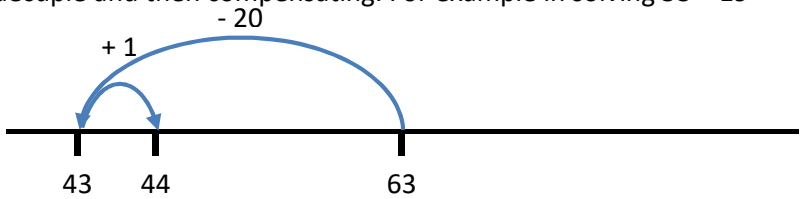
Jump Begin from one number, jump tens then ones or ones and then tens
E.g. 53 + 15



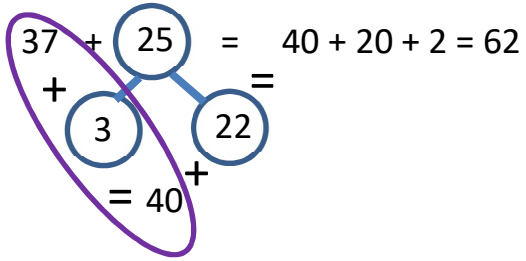


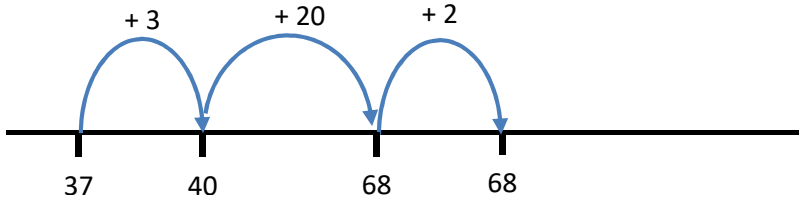
Over-jump Begin from one number, overshoot the jump and then compensate. To add or subtract 19, 18 or 17, jump 20, then back 1, 2, or 3. Over-jump is a form of compensation strategy, because it involves rounding one number to a decuple and then compensating. For example in solving 53 – 19



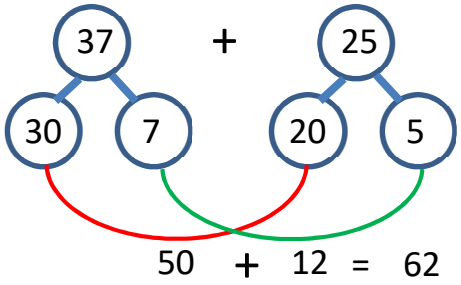


Jump to the decuple Begin from one number to jump to the nearest decuple, jump tens then jump remaining ones for example 37 + 25

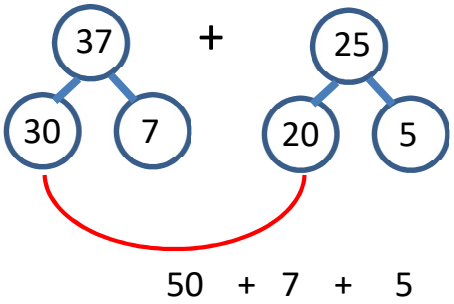


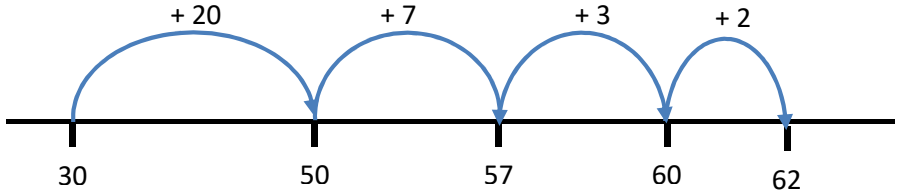


Split Working with the tens and ones separately and the re- combining, works well for non-regrouping tasks e.g 37 + 25
For subtraction with regrouping children can have significant difficulties e.g. 53-17 (they try 50-10, the 3-7)

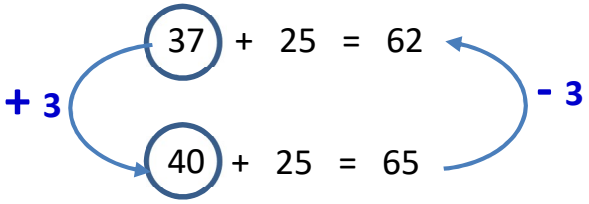


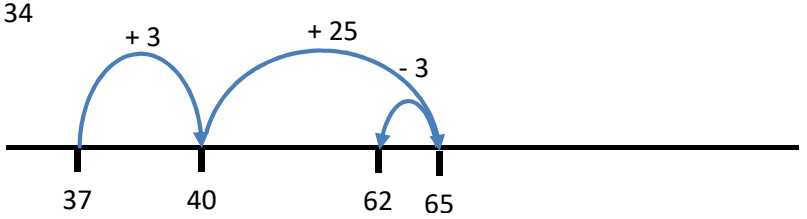
Split-jump Split tens and ones, add/subtract tens, add first ones, jump second ones. 37 + 25 involves firstly adding 30 + 20 to make 50 then add 7 makes 57 and to add 5 first add 3 to make 60 and 2 makes 62

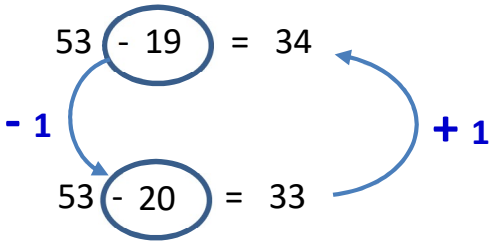


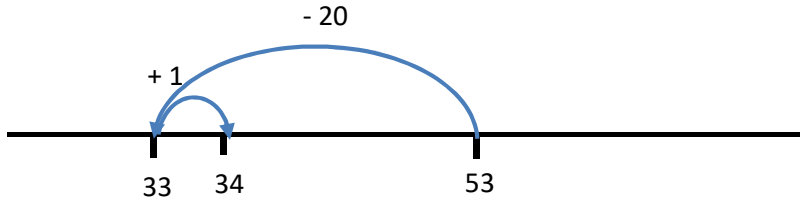


Compensation Adjusts one number to make an easier addition or subtraction, then does the calculation, and finally compensates for the initial adjustment
For 37 + 25: 40 + 25 → 65, 65 − 3 → 62 and for 53 − 19: 50 − 19 → 31 + 3 → 34

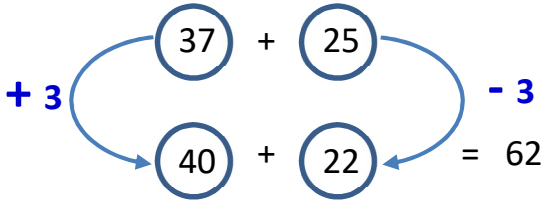


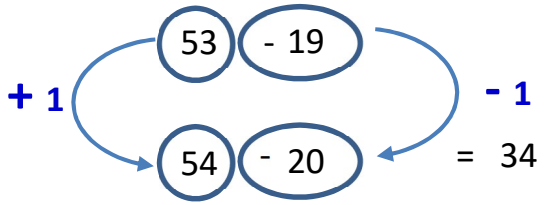




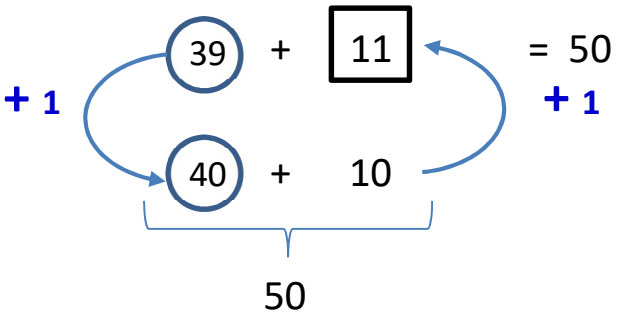


Transformation change both numbers to simplify an addition or subtraction while not altering the result, 37 + 25: change 37 to 40 and 25 to 22: 40 + 22= 62 and for 53 - 19: 54 – 20 = 34





Complementary addition or 'adding up to subtract' used especially when the difference between the minuend and subtrahend is relatively small. This strategy is likely to arise when a missing addend task is posed such as 39 + ☐ = 50. For 50 − 39, jump to the decuple is very common: 39 + 1 → 40, + 10 → 50, answer: 1 + 10 = 11. Commonly used for mentally calculating change in shopping transactions, this strategy is referred to as shopkeeper addition.



Jottings, Semi formal algorithms and formal algorithms

Writing can be used to make larger calculations more efficient and reliable. We distinguish three different kinds of written computation methods:
Jotting - for idiosyncratic informal writing to help solve a calculation problem.
Semi-formal written strategies are well-organised, standardised, usually taught written strategies.
Formal algorithms are probably the most familiar written computation methods. Algorithm means a step-by-step procedure for computing a standard task.

When children have developed competency with multi-digit mental computation, they are ready to develop written computation methods.
Children are invited to try jotting, and possibly invent their own written methods.
Over time, children become familiar with the practice of using writing to support calculations.
Semi-formal strategies can be introduced, in order of simplicity: addition first, then subtraction, and later, multiplication and division.
If children are continuing to need practical resources for calculating, consider whether they are ready for written methods.
Ensure the numbers given are appropriate to the strategy being used.

Jottings

Jottings focus on the development of efficient mental strategies for multiplication and division in the range 1 to 100. Next encourage them to try using jottings for tasks in the range 1 to 1000. The calculations are quite complex and jotting can become very useful. They may need help in organising their jottings. Children learning to connect their own writing with their own thinking is a fundamental goal.

Semi formal and Notating strategies

These are well organised strategies to help keep track of the calculation. They can be personal but more usually are modelled strategies which are embedded in a student's **number sense** and involves conceptual place value. It relies on multi-digit mental calculation and the student has a sense of what the answer might be.
Notating provides a record of the strategies children use to record their thinking after they have solved the task. As children describe their thinking, notate their strategies "How did you get that answer?"

Formal written strategies

These are referred to as *standard* or *traditional* strategies that show a step by step procedure dependent upon a precise layout. They rely on an understanding of conventional place value notation to keep track of the meaning. The final total is never conceived; instead it simply appears through the working out process.

Stage 1 - Early Years Foundation Stage

Number, Counting and Early Calculation

Although Maths is not a Prime area of the Foundation stage curriculum, as a school we are committed to ensuring children have a firm foundation in Maths to develop a secure and broad sense of Number, counting and early calculation.

Key Learning: Number

22- 36 months

Selects a small number of objects from a group when asked, for example, *'please give me one', 'please give me two'.*

- Recites some number names in sequence.
- Creates and experiments with symbols and marks representing ideas of number.
- Begins to make comparisons between quantities.
- Uses some language of quantities, such as *'more'* and *'a lot'.*
- Knows that a group of things changes in quantity when something is added or taken away.

30-50 months

- Uses some number names and number language spontaneously.
- Uses some number names accurately in play.
- Recites numbers in order to 10.
- Knows that numbers identify how many objects are in a set.
- Beginning to represent numbers using fingers, marks on paper or pictures.
- Sometimes matches numeral and quantity correctly.
- Shows curiosity about numbers by offering comments or asking questions.
- Compares two groups of objects, saying when they have the same number.
- Shows an interest in number problems.
- Separates a group of three or four objects in different ways, beginning to recognise that the total is still the same.
- Shows an interest in numerals in the environment.
- Shows an interest in representing numbers.
- Realises not only objects, but anything can be counted, including steps, claps or jumps.

40 – 60 months

- Recognise some numerals of personal significance.
- Recognises numerals 1 to 5.
- Counts up to three or four objects by saying one number name for each item.
- Counts actions or objects which cannot be moved.
- Counts objects to 10, and beginning to count beyond 10.
- Counts out up to six objects from a larger group.
- Selects the correct numeral to represent 1 to 5, then 1 to 10 objects.
- Counts an irregular arrangement of up to ten objects.
- Estimates how many objects they can see and checks by counting them.
- Uses the language of 'more' and 'fewer' to compare two sets of objects.
- Finds the total number of items in two groups by counting all of them.
- Says the number that is one more than a given number.
- Finds one more or one less from a group of up to five objects, then ten objects.
- In practical activities and discussion, beginning to use the vocabulary involved in adding and subtracting.
- Records, using marks that they can interpret and explain.
- Begins to identify own mathematical problems based on own interests and fascinations.

Early Learning Goal

Children count reliably with numbers from one to 20, place them in order and say which number is one more or one less than a given number. Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer. They solve problems, including doubling, halving and sharing.

Key Vocabualry: addition **add, more, and** **make, sum, total** **altogether** **score** **double** **one more, two more, ten more…** **how many more to make… ?** **how many more is… than…?**	**Key Vocabulary: subtraction** **take (away), leave** **how many are left/left over?** **how many have gone?** **one less, two less… ten less…** **how many fewer is… than…?** **difference between** **is the same as**
Skills in early addition Counting all – a child doing 2 and 3 more, counts out 2 bricks and then three bricks and then finds the total by counting all the bricks. Counting on from the first number – a child finding 3 + 5 counts on from the first number "four, five, six, seven, eight." Counting on from the larger number – a child chooses the larger number, even when it is not the first number and counts on from there. Using a known fact – where a child gives an immediate response to facts known by heart.	**Skills in early subtraction** Counting out – a child finding three less than 9 holds up 9 finger and folds 3 down. Counting back from – a child finding 9 take away 3 counts back three numbers from 9, "eight, seven, six." Counting back to – a child doing 7 – 3 counts back from the first number to the second, keeping a tally using fingers of the number of numbers that have been said, "six, five four."
Mental calculations Children must have strong number sense in order to develop early calculation skills. Haylock and Cockburn (2008) - Understanding number Cardinal aspect – matching a digit to the concrete objects, so matching the number 3 to 3 bears, counters, bikes… Nominal aspect – label various items and help distinguish items e.g. 3 on a bus. Ordinal aspect – to place things in order e.g. page 3 in a book, on a clock face, flat number 3. Counting Pre experiences ability to categorise - to be able to identify and separate off the members of a set in order that you can then count just these objects and no others. Rich experience of talk using language such as "one more" and "another one". Distinguish between small numbers before the engage in counting. Subitising. Stable order principle – repetition of saying numbers in order, it is fixed e.g. three always comes after 2. One to one principle – one number name to one object. Cardinal principle – the last number said is the number in the set. Ordinal and cardinal principle of number come together – to label the objects and order them. Abstraction principle – does not matter what the representation is for 3+4=7, you use the same number names, in the same order, with the same one to one matching process. Order irrelevance principal – whether count a row of objects forwards or backwards, it makes no difference. Conservation of number – arrangement of objects makes no difference.	

Early addition and subtractions skills to develop

On the boil

Autumn - Counting forwards and backwards to 10 by rote.
Find a numicon shape/dice shape which is one more/one less than numbers to 10.
Count objects accurately using one to one correspondence.

Spring - Find a numicon shape/dice shape which is one more/one less than teen numbers.
Count objects accurately using one to one correspondence.

Summer - Count on and back in 1s from any number up to 20. Say the number that is 1 more/less than a given number.
Use stories, pictures, objects to solve addition and subtraction problems up to 20. Practise counting on or back to find the answer.

Early multiplication and division skills to develop

On the boil

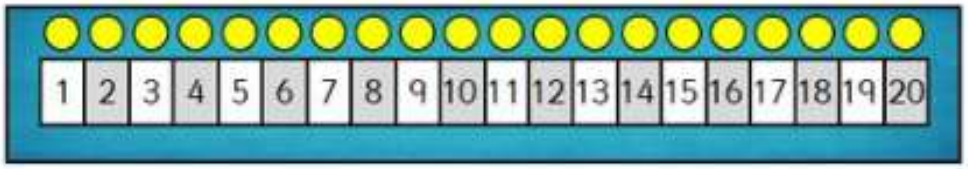
Autumn - Sharing objects between children/toys.

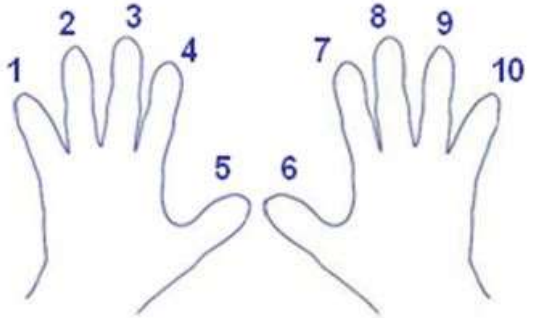
Spring - Sharing objects between children/toys and checking that it is an equal share (fair).

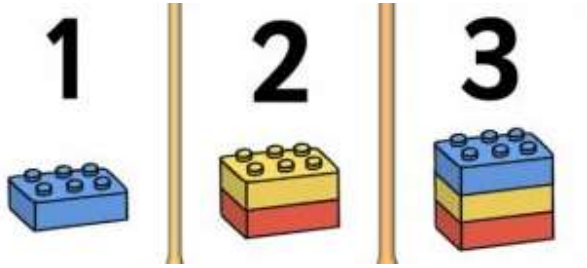
Summer - Show doubling using fingers or objects and using 'real life' situations e.g. double the number of children in the line, insects on the leaf. Chop objects in half and use language to describe what has happened. Half of a group of children to be inside and half to be outside.
Sharing objects between children/toys and checking that it is an equal share (fair).

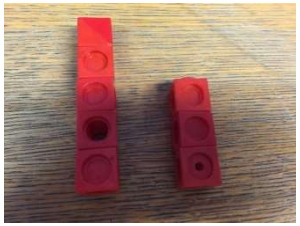
Visual Aids

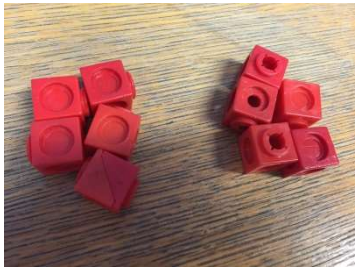
Number Tracks

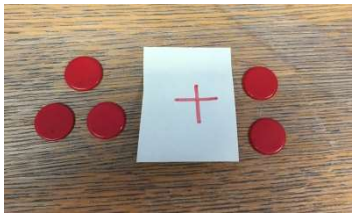












Stage 2 – Year 1 Addition and Subtraction

Key Learning:

- Solve problems involving counting, adding and subtracting in the context of numbers, measures or money, for example to 'pay' and 'give change'.
- Describe ways of solving puzzles and problems, explaining choices and decisions orally or using pictures.
- Read, write and interpret mathematical statements involving addition (+), subtraction (-) and equals (=) signs.
- Represent and use number bonds and related subtraction facts within 20.
- Add and subtract one-digit and two-digit numbers to 20, including zero (using concrete objects and pictorial representations) Relate addition to counting on; recognise that addition can be done in any order; use practical and informal written methods to support, understand subtraction as take away, find a difference by counting up; use practical and informal methods to support the subtraction.
- Solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as 7 = ☐ - 9.

Key Vocabulary:

+, add, more, **plus**
numberline
make, sum, total
altogether
score
double, **near double**
one more, two more… ten more
count on
how many more to make…?
how many more is… than…?
how much more is…?

Addition

Subtraction

-, subtract, take (away), **minus**
Leave
count back
how many are left/left over?
how many have gone?
one less, two less, ten less…
how many fewer is… than…?
how much less is…?
difference between
half, halve

=, equals, sign, is the same as

Mental calculations:

Count to and across 100, forwards and backwards, beginning with 0 or 1, or from any given number.
Count in multiples of twos, fives and tens.
Given a number, identify one more and one less for multiples of 10.

On the boil

Use stories, pictures, objects to build up the story of all numbers to 5/10.
Find totals by counting all.
To use the equals sign to balance sums using apparatus.
Find totals of two sets of objects by counting on.
Doubling and halving objects up to double 10.

Addition – Add with numbers up to 20.

Children should:

Have access to a wide range of counting equipment, everyday objects, number tracks and number lines, and be shown numbers in different contexts.

Read and write the addition (+) and equals (=) signs within number sentences.

Interpret addition number sentences and solve missing box problems, using concrete objects and number line addition to solve them:

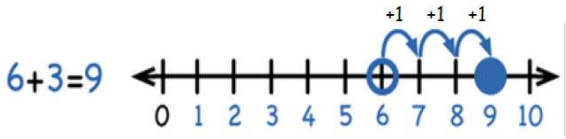
8 + 3 = □ 5 + 4 = □ 5 + 3 + 1 = □ □ + □ = 6

This builds on from prior learning of adding by combining two sets of objects into one group (5 cubes and 3 cubes) in Early Years.

The four advanced counting strategies –

Counting on e.g. 8 + 4 = [ ]
Counting up to e.g. 8 + [ ] = 12
Counting back from 12 - 4 = [ ]
Counting back to 12- [ ] = 8.

Use numbered number lines to add, by counting on in ones. Encourage children to start with the larger number and count on.



Numberlines, bead strings or bead bars can be used to illustrate addition, including bridging though ten by counting on 2 then on 3.

8 + 5 =

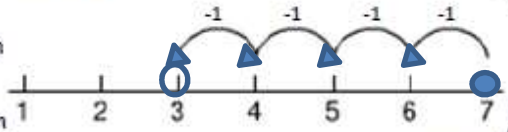
Subtraction – Subtract from numbers up to 20.

Children consolidate understanding of subtraction practically, showing subtraction on bead strings, using cubes etc. and in familiar contexts, and are introduced to more formal recording using number lines as below:

Read, write and interpret number sentences with - and = signs.

Subtract by taking away

Count back in ones on a numbered number line to take away, with numbers up to 20:

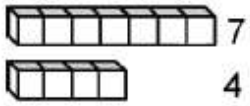


7 – 4 = 3

Model subtraction using hundred squares and numbered number lines/tracks and practically.

Find the distance / difference

This will be introduced practically with the language '**find the distance between**' and '**how many more?**' in a range of familiar contexts.



'Seven is 3 more than four'

'I am 2 years older than my sister'

Problems should include the terms: put together, add, altogether, total, take away, distance between, difference between, more than and less than, so that pupils develop the concept of addition and subtraction and are enabled to use these operations flexibly.

Stage 2 – Year 2 Addition and Subtraction

Key Learning:

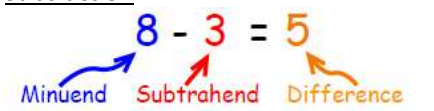
- Present solutions to puzzles and problems in an organised way; explain decisions, methods and results in pictorial, spoken or written form, using mathematical language and number sentences.
- Solve problems involving addition and subtraction, multiplication in contexts of numbers, measures or pounds and pence.
- Identify and record the information or calculation needed to solve a puzzle or problem; carry out the steps or calculations and check the solution in the context of the problem.
- Choose an appropriate strategy to solve a calculation based upon the numbers involved (recall a known fact, calculate mentally, use a jotting).
- Select a mental strategy appropriate for the numbers involved in the calculation.
- Show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot.
- Understand subtraction as take away and difference (how many more, how many less/fewer).
- Recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100.
- Recall and use number bonds for multiples of 5 totalling 60 (to support telling time *to nearest 5 minutes).*
- Add and subtract numbers using concrete objects, pictorial representations, and mentally, including:
    - a two-digit number and ones (units)
    - a two-digit number and tens
    - two two-digit numbers
    - adding three one-digit numbers

Key Vocabulary:

+, add, **addition**, more, plus
make, sum, total, altogether
score, double, near double
one more, two more... ten more... one hundred more
how many more to make…?
how many more is… than…?
how much more is…?

Addition

Subtraction



-, subtract, **subtraction,** take (away), minus
leave, how many are left/left over?
one less, two less… ten less… one hundred less
how many fewer is… than…?
how much less is…?
difference between
half, halve

=, equals, sign, is the same as
tens boundary

Mental calculations:

Count in steps of 2, 3, and 5 from 0, and in 10s from any number, forward and backward.
Find 1 or 10 more or less than a given number.

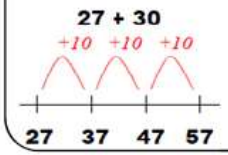
On the boil - Autumn	**On the boil - Spring**	**On the boil - Summer**
Recalling addition and subtraction facts to 10 and 20. Addition and subtraction of single-digit numbers, including crossing 10 by counting on/back. Add three single digit numbers by reordering i.e. 1 + 3 + 9 – putting the largest number 1st or finding bonds. Doubling and halving numbers – rapid recall up to 10+10.	Recalling addition and subtraction facts within 20, interpreting missing number questions with = symbol in any position. Addition and subtraction of teen numbers and single digit numbers by counting on/back from the largest number. Add several single digit numbers by reordering i.e. 3 + 3 + 8 + 2 = using doubles/bonds. Doubling and halving numbers – rapid recall up to 10 + 10.	Recalling addition and subtraction facts within 20, represented as missing number problems with = symbol in any position. Addition and subtraction of two digit and single digit numbers by reordering i.e. putting largest number first/looking for bonds. Add several single digit numbers by reordering i.e. 3 + 3 + 8 + 2 = using doubles/bonds. Doubling and halving numbers – rapid recall up to 20 + 20.

Addition – Add with 2-digit numbers.

Developing mental fluency with addition and place value involving 2-digit numbers, then establish more formal methods.

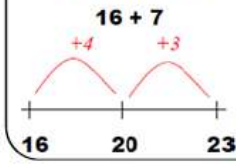
Add 2-digit numbers and tens:

27 + 30



Add 2-digit numbers and units:

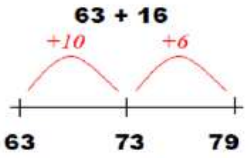
16 + 7



Use empty number lines, concrete equipment, hundred squares etc. to build confidence and fluency in mental addition skills.

Add pairs of 2-digit numbers, moving to the partitioned column method when secure adding tens and units:

63 + 16



23 + 34:

20	+	3	
+ 30	+	4	
50	+	7	
	=	57	

STEP 1: Only provide examples that do NOT cross the tens boundary until they are secure with the method itself.

STEP 2: Once children can add a multiple of ten to a 2-digit number mentally (e.g. 80+11), they are ready for adding pairs of 2-digit numbers that DO cross the tens boundary (e.g. 58 + 43).

58 + 43:

50	+	8	
40	+	3	
90	+	11	
	=	101	

To support understanding, pupils may physically make and carry out the calculation with arrow cards, place value rods or place value counters, then compare their practical version to the written form, to help them to build an understanding of it.

Subtraction – subtract with 2-digit numbers

Subtract on a number line by counting back, aiming to develop mental subtraction skills.

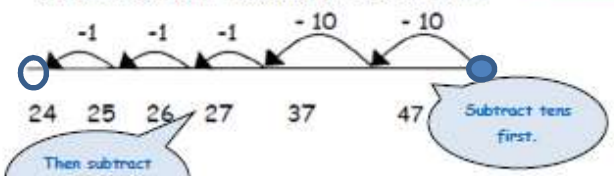
Use Dienes blocks for subtraction calculations too.

This strategy will be used for:

- **2-digit numbers subtract units** (by taking away / counting back) e.g. 36—7
- **2-digit numbers subtract tens** (by taking away / counting back) e.g. 48—30
- **Subtracting pairs of 2-digit numbers** (see below:)

Subtracting pairs of 2-digit numbers on a number line:

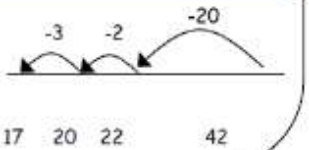
47 - 23 = 24 Partition the second number and subtract it in tens and units, as below:



Move towards more efficient jumps back, as below:

Combine methods with use of a hundred square to reinforce understanding of number value and order.

Teaching children to **bridge through ten** can help them to become more efficient, for example 42—25:



Stage 3 – Year 3 Addition and Subtraction

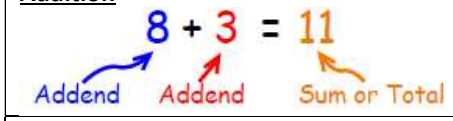
Key Learning:

- Recall/use addition/subtraction facts for 100 (multiples of 5 and 10).
- Derive and use addition and subtraction facts for 100.
- Derive and use addition and subtraction facts for multiples of 100 totalling 1000.
- Add and subtract numbers mentally, including:
  - a three-digit number and ones (units)
  - a three-digit number and tens
  - a three-digit number and hundreds
- Add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction.
- Estimate the answer to a calculation and use inverse operations to check answers.
- Solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction.

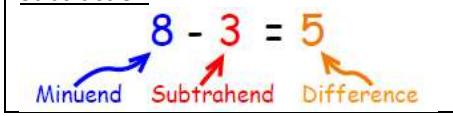
Key Vocabulary:

+, add, addition, more, plus
make, sum, total
altogether
score
double, near double
one more, two more... ten more... one hundred more
how many more to make…?
how many more is… than…?
how much more is…?

Addition



Subtraction



-, subtract, subtraction, take (away), minus
leave, how many are left/left over?
one less, two less… ten less… one hundred less
how many fewer is… than…?
how much less is…?
difference between
half, halve

=, equals, sign, is the same as
tens boundary, **hundreds boundary**

Mental calculations:

Find *1,* 10 or 100 more or less than a given number.
Count up and down in tenths.

On the boil

Rapid recall of addition and subtraction facts within 20, represented as missing number problems with = symbol in any position.
Doubling and halving numbers – rapid recall up to 20 + 20.

Autumn	**Spring**	**Summer**
Rapid recall of pairs of numbers totalling 5, 10 and 20. Bridging when adding a single digit to a two digit number. Ie. 36 + 7 = 36 + 4 + 3. Add two/three numbers by reordering ie. 6 + 7 + 3 =.	Use bonds to derive pairs of numbers to any multiple of 10 or 100. Ie. 3 + 7 = 10 therefore 13 + 7 = 20 and 23 + 7 = 30. Bridging/rounding when adding a single digit number to a two or three digit number ie. 136 + 8 = 136 + 4+ 4 or 136 + 10 -2. Add two/three numbers by reordering ie. 6 + 7 + 3 =. Use doubles to support understanding of 4s and 8s times tables.	Use bonds to derive pairs of numbers to any multiple of 10 or 100 i.e. 3 + 7 = 10 therefore 13 + 7 = 20 and 23 + 7 = 30. Bridging/rounding when adding a single digit number to a two or three digit number i.e. 136 + 8 = 136 + 4+ 4 or 136 + 10 -2. Add two/three two digit numbers by reordering ie. 40 + 25 + 60 + 75 =. Use doubles to support understanding of 4s and 8s times tables.

Addition – Add numbers with up to 3-digit.

Introduce the **expanded column addition** method:

	2	3	6
+		7	3
			9
	1	0	0
	2	0	0
	3	0	9

Add the **units** first, in preparation for the compact method.

In order to carry out this method of addition:

- Children need to recognise the value of the hundreds, tens and units without recording the partitioning.
- Pupils need to be able to add in columns.

Move to the compact **column addition** method, with 'carrying':

Add **units** first.

'Carry' numbers **underneath** the bottom line.

```
 236
+ 73
 309
  1
```

Children who are very secure and confident with 3-digit expanded column addition should be moved onto the **compact column addition** method, being introduced to 'carrying' for the first time. Compare the expanded method to the compact column method to develop an understanding of the process and the reduced number of steps involved.

Remind pupils the actual value is '**three tens** add **seven tens**', not 'three add seven', which equals **ten** tens.

Subtraction – Subtracting with 2 and 3-digit numbers.

Introduce **partitioned column subtraction** method.

STEP 1; introduce this method with examples where **no exchanging** is required.

89 - 35 = 54

80 + 9
- 30 + 5
50 + 4

When learning to 'exchange', explore 'partitioning in different ways' so that pupils understand that when you exchange, the **VALUE** is the same ie **72** = 70+2 = 60+12 = 50+22 etc. Emphasise that the **value hasn't changed**, we have just partitioned it in a different way.

STEP 2; introduce 'exchanging' through practical subtraction. Make the larger number with Base 10, then subtract 47 from it.

72 - 47

60 70 + 12
- 40 + 7
20 + 5 = **25**

Before subtracting '7' from the 72 blocks, they will need to exchange a row of 10 for ten units. Then subtract 7, and subtract 4 tens.

STEP 3; Once pupils are secure with the understanding of 'exchanging', they can use the partitioned column method to subtract any 2 and 3-digit numbers.

238 - 146 = 92

100
200 + 130 + 8
- 100 + 40 + 6
0 + 90 + 2

Stage 3 – Year 4 Addition and Subtraction

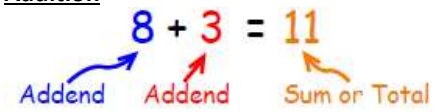
Key Learning:

- Report solutions to puzzles and problems, giving explanations and reasoning orally and in writing, using diagrams and symbols.
- Choose an appropriate strategy to solve a calculation based upon the numbers involved (recall a known fact, calculate mentally, use a jotting, written method).
- Select a mental strategy appropriate for the numbers involved in the calculation.
- Recall and use addition and subtraction facts for 100.
- Recall and use +/- facts for multiples of 100 totalling 1000.
- Derive and use addition and subtraction facts for 1 and 10 (with decimal numbers to one decimal place).
- Add and subtract mentally combinations of two and three digit numbers and decimals to one decimal place.
- Add and subtract numbers with up to 4 digits and decimals with one decimal place using the formal written methods of columnar addition and subtraction where appropriate.
- Estimate; use inverse operations to check answers to a calculation.
- Solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why.
- Solve addition and subtraction problems involving missing numbers.

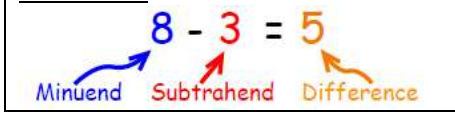
Key Vocabulary:

add, addition, more, plus, **increase**
sum, total, altogether
score
double, near double
how many more to make…?

Addition



Subtraction



subtract, subtraction, take (away), minus, **decrease**
leave, how many are left/left over?
difference between
half, halve
how many more/fewer is… than…?
how much more/less is…?

equals, sign, is the same as
tens boundary, hundreds boundary
inverse

Mental calculations:
Count in multiples of 6, 7, 9, 25 and 1000.

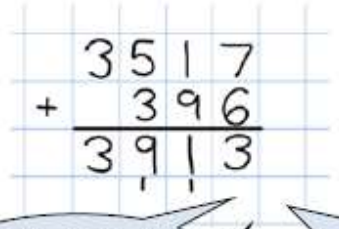
On the boil
Rapid recall of addition and subtraction facts within 20, represented as missing number problems with = symbol in any position. Make links to finding facts to 200, 2000 etc.,
Children practice selecting which mental calculation strategy is the most efficient when presented with different calculations i.e. round and adjust, find the difference, reorder, partition, count on, count back, doubles, near doubles, halves and bonds.

Autumn	**Spring**	**Summer**
Use bonds to derive pairs of numbers to any multiple of 10 or 100. Ie. 3 + 7 = 10 therefore 13 + 7 = 20 and 23 + 7 = 30.	Derive bonds to 1 etc from bonds to 10. Ie. 3 + 7 = 10 therefore 0.3 + 0.7 = 1.	Derive bonds to 0.1, etc from known bonds ie. 4 + 6 = 10 therefore 0.04 + 0.06 = 0.1.

Addition – Add with numbers up to 4-digit.

Move from expanded addition to the compact column method, **adding units first**, and 'carrying' numbers **underneath** the calculation. Also include money and measures contexts.

e.g. 3517 + 396 = 3913



Introduce the **compact column addition** method by asking children to add the two given numbers together using the method that they are familiar with (expanded column addition—see Y3). Teacher models the compact method with carrying, asking children to discuss similarities and differences and establish how it is carried out.

Add **units** first.

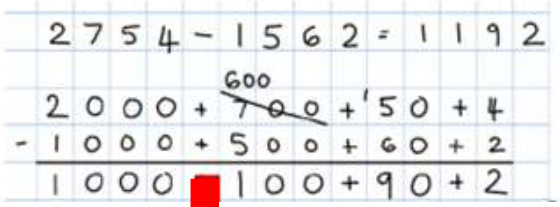
'Carry' numbers **underneath** the bottom line.

Reinforce correct place value by reminding them the actual value is 5 hundreds add 3 hundreds, **not 5 add 3**, for example.

Use and apply this method to money and measurement values.

Subtraction – Subtract with up to 4-digit numbers.

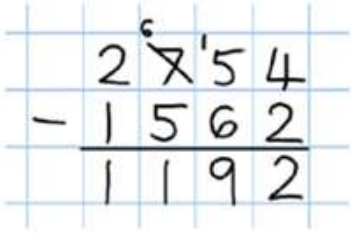
Partitioned column subtraction with 'exchanging' (decomposition):



As introduced in Y3, but moving towards more complex numbers and values. Use **place value counters** to reinforce 'exchanging'.

Children who are still not secure with number facts and place value will need to remain on the partitioned column method until ready for the compact method.

Compact column subtraction



To introduce the compact method, ask children to perform a subtraction calculation with the familiar partitioned column subtraction then display the compact version for the calculation they have done. Ask pupils to consider how it relates to the method they know, what is similar and what is different, to develop an understanding of it (shown on video).

Give plenty of opportunities to apply this to money and measures.

Always encourage children to consider the best method for the numbers involved—mental, counting on, counting back or written method (see video).

Stage 4 – Year 5 Addition and Subtraction

Key Learning:

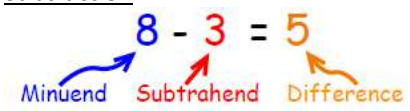
- Choose an appropriate strategy to solve a calculation based upon the numbers involved (recall a known fact, calculate mentally, use a jotting, written method).
- Select a mental strategy appropriate for the numbers involved in the calculation.
- Recall and use addition and subtraction facts for 1 and 10 (with decimal numbers to one decimal place).
- Derive and use addition and subtraction facts for 1 (with decimal numbers to two decimal places).
- Add and subtract numbers mentally with increasingly large numbers and decimals to two decimal places.
- Add and subtract whole numbers with more than 4 digits and decimals with two decimal places, including using formal written methods (columnar addition and subtraction).
- Use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy.
- Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.
- Solve addition and subtraction problems involving missing numbers.

Key Vocabulary:

add, addition, more, plus, increase
sum, total, altogether
score
double, near double
how many more to make…?

Addition

Subtraction



subtract, subtraction, take (away), minus, decrease
leave, how many are left/left over?
difference between
half, halve
how many more/fewer is… than…?
how much more/less is…?

equals, sign, is the same as
tens boundary, hundreds boundary
units boundary, tenths boundary
inverse

Mental calculations:

Count forwards or backwards in steps of powers of 10 for any given number up to 1 000 000.
Count forwards and backwards in decimal steps.

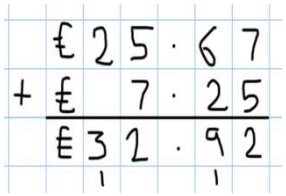
On the boil

Children practice selecting which mental calculation strategy is the most efficient when presented with increasingly challenging calculations i.e. round and adjust, find the difference, reorder, partition, count on, count back, doubles, near doubles, halves and bonds.

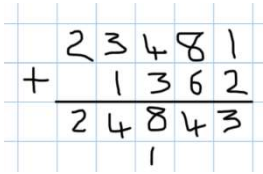
Autumn	**Spring**	**Summer**
Derive bonds to 0.1, etc from known bonds i.e. 4 + 6 = 10 therefore 0.04 + 0.06 = 0.1.	Derive bonds to hundredths from known bonds i.e. 4 + 6 = 10 therefore 0.004 + 0.006 = 0.01.	Derive as many facts as possible from bonds to 10 using place value knowledge.

Addition – Add numbers with more than 4 digits.

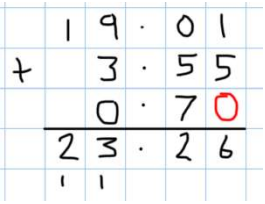
This needs to include money, measures and decimals with different numbers of decimal places.



The decimal point should be aligned in the same way as the other place value colums and must be in the same column in the answer.



Numbers should exceed 4 digits.



Pupils should be able to add more than two values, careful alligning place value colums.

Empty decimal places can be filled with zero to show the place

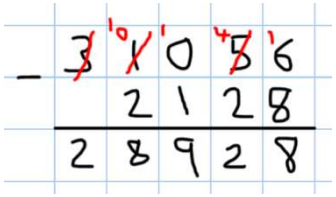
Children should understand the place value of tenths and hundredths and use this to align numbers with different numbers of decimal places

Subtraction – Subtract with at least 4-digit numbers.

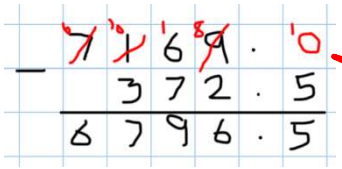
This needs to include money, measures and decimals with different numbers of decimal places.

Compact column subtraction (with exchanging / borrowing)

Children who are still not secure with number facts and place value will need to remain on the partitioned column method until ready for the compact method.



Subtracting with larger integers.



Subtract with decimal values, including mixtures of integers and decimals, aligning the decimal point

Empty decimal places can be filled with zero to show the place

Create lots of opportunities for subtracting and finding differences with money and measures.

Stage 4 – Year 6 Addition and Subtraction

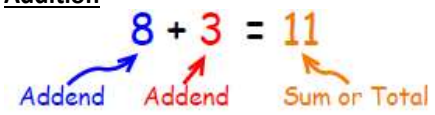
Key Learning:

- Choose an appropriate strategy to solve a calculation based upon the numbers involved (recall a known fact, calculate mentally, use a jotting, written method).
- Select a mental strategy appropriate for the numbers in the calculation.
- Recall and use addition and subtraction facts for 1 (with decimals to two decimal places).
- Perform mental calculations including with mixed operations and large numbers and decimals.
- Add and subtract whole numbers and decimals using formal written methods (columnar addition and subtraction).
- Use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy.
- Use knowledge of the order of operations to carry out calculations.
  Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.
- Solve problems involving all four operations, including those with missing numbers.

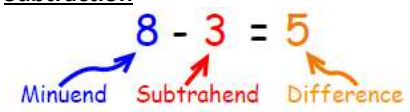
Key Vocabulary:

add, addition, more, plus, increase
sum, total, altogether
score
double, near double
how many more to make…?

Addition



Subtraction



subtract, subtraction, take (away), minus, decrease
leave, how many are left/left over?
difference between
half, halve
how many more/fewer is… than…?
how much more/less is…?

equals, sign, is the same as
tens boundary, hundreds boundary
units boundary, tenths boundary
inverse

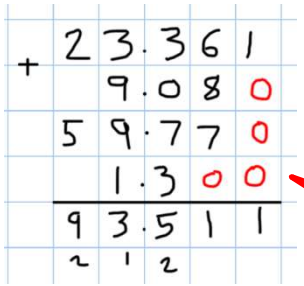
Mental calculations:

Count forwards or backwards in steps of integers, decimals, powers of 10.
Choose an appropriate strategy to solve a calculation based upon the numbers involved (recall a known fact, calculate mentally, use a jotting, written method).
Select a mental strategy appropriate for the numbers in the calculation.
Recall and use addition and subtraction facts for 1 (with decimals to two decimal places).
Perform mental calculations including with mixed operations and large numbers and decimals.

On the boil

Rapid recall of addition and subtraction facts within 20, represented as missing number problems with = symbol in any position. Make links to algebra i.e. 2x + 3 = 20.
Derive as many facts as possible from bonds to 10/20 using place value knowledge i.e. 120 + 80 = 200 because 12 + 8 = 20 and 3.1 + 0.9 = 4 because one tenth and nine tenths equal ten tenths or one. Make links to algebra 5 + ? = 10.6.
Children practice selecting which mental calculation strategy is the most efficient when presented with increasingly challenging calculations i.e. round and adjust, find the difference, reorder, partition, count on, count back, doubles, near doubles, halves and bonds.

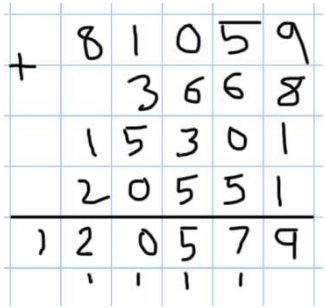
Addition – Add several numbers of increasing complexity.



Adding several number with different numbers of deciaml places, including money and measures.

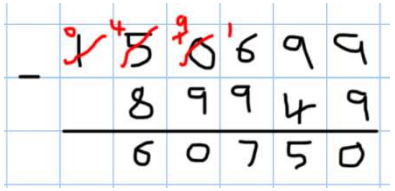
Decimal points can now be on the line and do not need a separate box fo their own.

Empty decimal places can be filled with zero to show the place

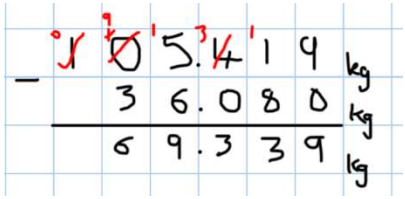


Adding several numbers with more than 4 digits

Subtraction – Subtracting with increasingly larger and more complex numbers and decimal values.



Using the compact method to subtract more complex integers.



Using the compact column method to subtract money and measures, including decimals with different numbers of decimal places.

Pupils should be able to apply their knowledge of a range of mental strategies, mental recall skills, and informal and formal written methods when selecting the **most appropriate method** to work out subtraction problems.

Stage 2 – Year 1 Multiplication and Division

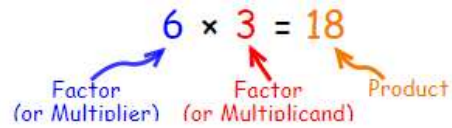
Key Learning:

- Recall and use doubles of all numbers to 10 and corresponding halves.
- Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

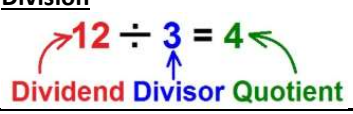
Key Vocabulary:

double

Multiplication



Division



halve

share

left, left over

Mental calculations:

Count to and across 100, forwards and backwards, beginning with 0 or 1, or from any given number

Count in multiples of twos, fives and tens

Recognise and create repeating patterns with numbers, objects and shapes

On the boil

Autumn

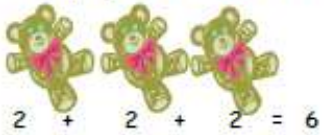
Grouping and sharing contexts

Spring and Summer

Creating arrays to explore the vocabulary around multiplication and division.

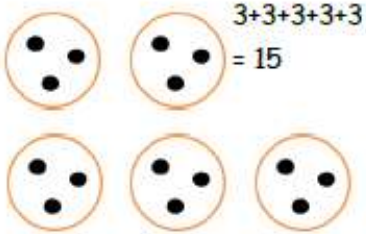
Multiplication – Multiply with concrete objects, arrays and pictorial representations.

How many legs will 3 teddies have?



2 + 2 + 2 = 6

There are 3 sweets in one bag.
How many sweets are in 5 bags altogether?



3+3+3+3+3
= 15

- Give children experience of counting equal group of objects in 2s, 5s and 10s.
- Present practical problem solving activities involving counting equal sets or groups, as above.

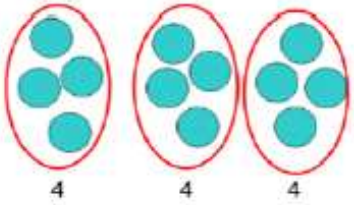
Division – Group and share small quantities.

Using objects, diagrams and pictorial representations to solve problems involving **both grouping and sharing.**

How many groups of 4 can be made with 12 stars? = 3

Grouping:

Sharing:



4 4 4

12 shared between 3 is 4

Example division problem in a familiar context:

There are 6 pupils on this table and there are 18 pieces of fruit to share between us. If we share them equally, how many will we each get?

Can they work it out and give a division statement... ?

"18 shared between 6 people gives you 3 each."

Pupils should :

- use lots of practical apparatus, arrays and picture representations
- Be taught to understand the difference between 'grouping' objects (How many groups of 2 can you make?) and 'sharing' (Share these sweets between 2 people)
- Be able to count in multiples of 2s, 5s and 10s.
- Find **half** of a group of objects by sharing into 2 equal groups.

Stage 2 – Year 2 Multiplication and Division

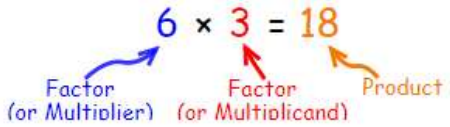
Key Learning:

- Understand multiplication as repeated addition.
- Understand division as sharing and grouping and that a division calculation can have a remainder.
- Show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot.
- Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers.
- Derive and use doubles of simple two-digit numbers (numbers in which the ones total less than 10).
- Derive and use halves of simple two-digit even numbers (numbers in which the tens are even).
- Calculate mathematical statements for multiplication using repeated addition) and division within the multiplication tables and write them using the multiplication (×), division (÷) and equals (=) signs.
- Solve problems involving multiplication and division (including those with remainders), using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.

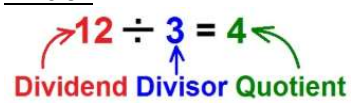
Key Vocabulary:

lots of, groups of
´, times, multiply, multiplied by
multiple of
once, twice, three times… ten times…
times as (big, long, wide… and so on)
repeated addition
array
row, column
double

Multiplication



Division



halve
share, **share equally**
one each, two each, three each…
group in pairs, threes… tens
equal groups of…
divide, divided by, divided into
left, left over

Mental calculations:

Count in steps of 2, 3, and 5 from 0, and in tens from any number, forward and backward.
Understand the connection between the 10 multiplication table and place value.

On the boil

Autumn

Rapid recall of multiplication and related division facts – 2s and 10s link to arrays and jumps along a number line.

Spring

Rapid recall of multiplication and related division facts – 2s, 5s and 10s link to arrays and jumps along a number line.

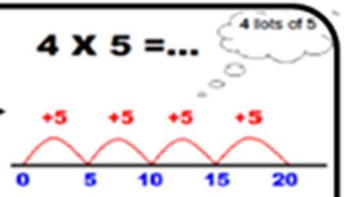
Summer

Rapid recall of multiplication and related division facts – 2s, 5s and 10s link to arrays and jumps along a number line.

Multiplication – Multiply using arrays and repeated addition (using at least 2s, 5s and 10s).

Use repeated addition on a number line:

- Starting from zero, make equal jumps up on a number line to work out multiplication facts and write multiplication statements using x and = signs.



Use arrays:

5 x 3 = 15

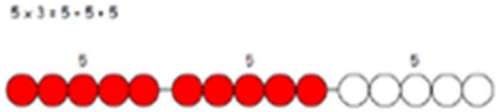
5 x 3 = 3 + 3 + 3 + 3 + 3 = 15

3 x 5 = 5 + 5 + 5 = 15

3 x 5 = 15

Use arrays to help teach children to understand the commutative law of multiplication, and give examples such as 3 x __ = 6.

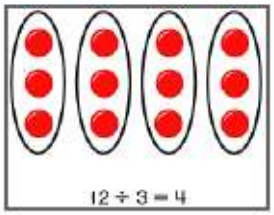
Use practical apparatus:



Division – Group and share small, using the ÷and = sign.

Use objects, arrays, diagrams and pictorial representations, and grouping on a number line.

Arrays:

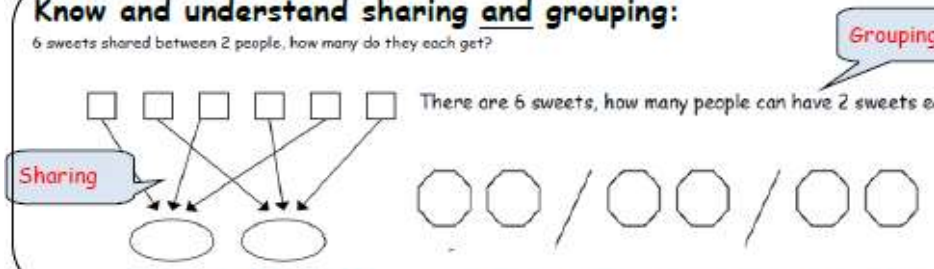


This represents 12 ÷ 3, posed as how many groups of 3 are in 12?

Pupils should also show that the same array can represent 12 ÷ 4 = 3 if grouped horizontally.

Know and understand sharing and grouping:

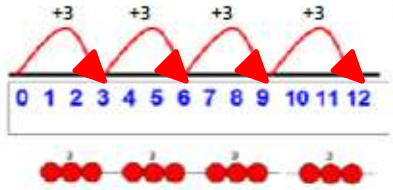
6 sweets shared between 2 people, how many do they each get?



Children should be taught to recognise whether problems require sharing or grouping.

Grouping using a number line:

Group from zero in equal jumps of the divisor to find out 'how many groups of _ in _ ?'. Pupils could and using a bead string or practical apparatus to work out problems like 'A CD costs £3. How many CDs can I buy with £12?' **This is an important method to develop understanding of division as grouping.**



12 ÷ 3 = 4

Pose 12 ÷ 3 as 'How many groups of 3 are in 12?'

Stage 3 – Year 3 Multiplication and Division

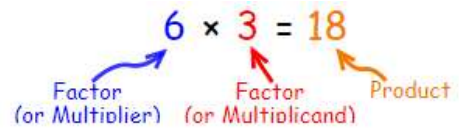
Key Learning:

- Understand that division is the inverse of multiplication and vice versa.
- Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables.
- Derive and use doubles of all numbers to 100 and corresponding halves.
- Derive and use doubles of all multiples of 50 to 500.
- Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods.
- Use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy.
- Solve problems, including missing number problems, involving multiplication and division *(and interpreting remainders),* including positive integer scaling problems and correspondence problems in which n objects are connected to m objects.

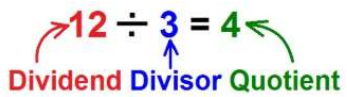
Key Vocabulary:

lots of, groups of
´, times, multiply, **multiplication,**
multiplied by
multiple of, **product**
once, twice, three times… ten times…
times as (big, long, wide… and so on)
repeated addition
array
row, column
double,

Multiplication



Division



halve
share, share equally
one each, two each, three each…
group in pairs, threes… tens
equal groups of
„, divide, **division,** divided by, divided into
left, left over, **remainder**

Mental calculations:

Count from 0 in multiples of 4, 8, 50 and 100.
Count up and down in tenths.
Find the effect of multiplying a one- or two-digit number by 10 and 100, identify the value of the digits in the answer

Summer	**Autumn**	**Spring**
Rapid recall of multiplication and related division facts – 1, 2, 3, 4, 5, 8, 50 and 100.	Rapid recall of multiplication and related division facts – 2s, 5s and 10s.	Rapid recall of multiplication and related division facts – 2s, 5s, 4s, 50s, 100s.

Multiplication – Multiply 2-digits by a single digit number.

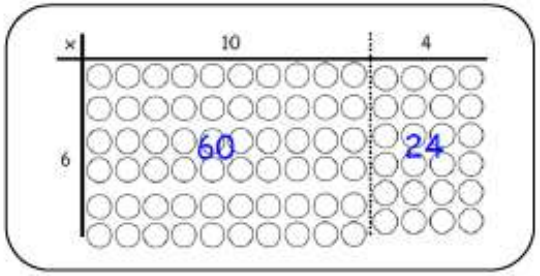
Introduce the grid method for multiplying 2-digit by single-digits:

Eg. 23 x 8 = 184

X	20	3
8	160	24

160 + 24 = 184

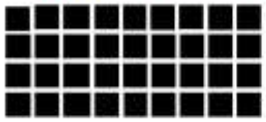
Link the layout of the grid to an array initially:



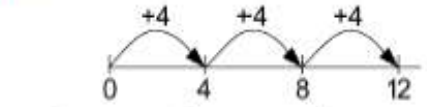
Introduce the grid method with children physically making an array to represent the calculation (e.g. make 8 lots of 23 with 10s and 1s place value counters), then translate this to grid method format (see video clip).

To do this, children must be able to:

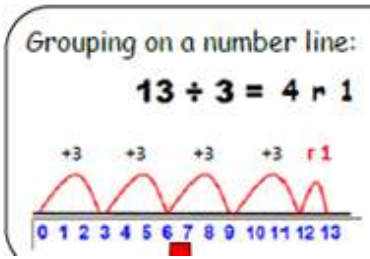
- Partition numbers into tens and units
- Multiply multiples of ten by a single digit (e.g. 20 x 4) using their knowledge of multiplication facts and place value
- Recall and work out multiplication facts in the **2, 3, 4, 5, 8 and 10** times tables.
- Work out multiplication facts not known by repeated addition or other taught mental strategies (e.g. by commutative law, working out near multiples and adjusting, using doubling etc.) Strategies to support this are repeated addition using a number line, bead bars and arrays:



9 x 4 = 36



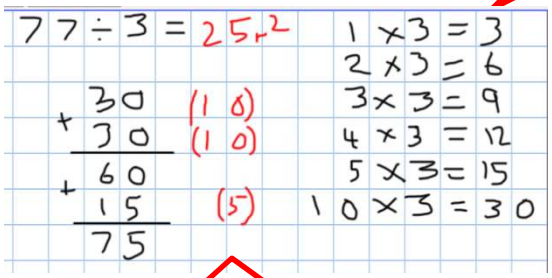
Division – Divide 2-digit numbers by a single digit



STEP 1: Children continue to work out unknown division facts by grouping on a number line from zero. They are also now taught the concept of remainders, as in the example. This should be introduced practically and with arrays, as well as being translated to a number line. Children should work towards calculating some basic division facts with remainders mentally for the 2s, 3s, 4s, 5s, 8s and 10s, ready for 'carrying' remainders across within the short division method.

Create a 'partial tables' grid first.

Division using 'Chunking Up'



Put the multiple you are adding in brackets. You add these up at the end.

Stage 3 – Year 4 Multiplication and Division

Key Learning:

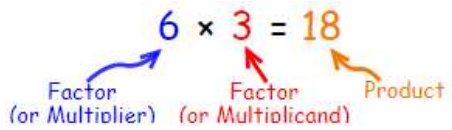
- *Choose an appropriate strategy to solve a calculation based upon the numbers involved (recall a known fact, calculate mentally, use a jotting, written method).*
- Recognise and use factor pairs and commutativity in mental calculations.
- Recall multiplication and division facts for multiplication tables up to 12 × 12.
- *Use partitioning to double or halve any number, including decimals to one decimal place.*
- Use place value, known and derived facts to multiply and divide mentally, including:
- multiplying by 0 and 1
- dividing by 1
- multiplying together three numbers
- Multiply two-digit and three-digit numbers by a one-digit number using formal written layout.
- *Divide numbers up to 3 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context.*
- *Use estimation and inverse to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy.*
- Solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, *division (including interpreting remainders),* integer scaling problems and harder correspondence problems such as n objects are connected to m objects.

Key Vocabulary:

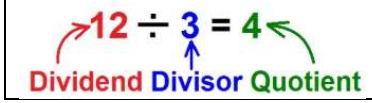
lots of, groups of
times, multiply, multiplication, multiplied by
multiple of, product

once, twice, three times… ten times…
times as (big, long, wide… and so on)
repeated addition
array column
double,
row,

Multiplication



Division



halve
share, share equally
one each, two each, three each…
group in pairs, threes… tens
equal groups of
divide, division, divided by, divided into
remainder
factor, quotient, divisible by
inverse

Mental calculations:
Count in multiples of 6, 7, 9, 25 and 1000.

On the boil - Autumn	**On the boil - Spring**	**On the boil - Summer**
Doubling and halving numbers – rapid recall up to 20 + 20. Rapid recall of multiplication and related division facts – 1, 2, 3, 4, 5, 8, 50 and 100.	Double and halve two and three digit numbers by partitioning. Rapid recall of 11 and 9 times tables (if learnt in Autumn term).	Using double facts to double and halve tenths. Rapid recall of all multiplication facts up to 12 x 12.

Multiplication – Multiply 2 and 3-digits by a single digit, using all multiplication tables up to 12 x 12.

Developing the grid method

x	100	30	5
4	400	120	20

400 + 120 + 20 = 540

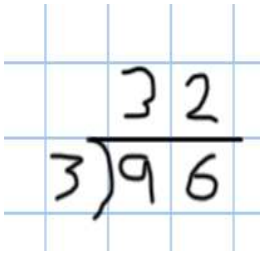
Encourage column addition to add accurately

Children should be able to:

Approximate before they calculate, and make this a regular part of their calculating, going back to the approximation to check the reasonableness of their answer. Eg:
346 x 9 is approximately 350 x 10 = 3500
Multiply multiples of ten and one hundred by a single-digit, using their multiplication table knowledge.
Recall all times tables up to 12 x 12.

Division – Divide 2 and 3 digit numbers by a single digit, with no remainders in the final answer.

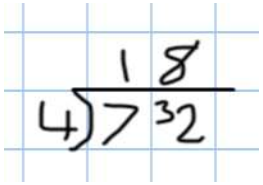
'Bus-stop' method for division



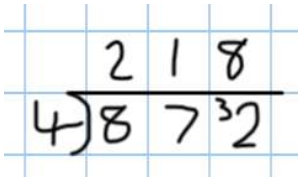
Limited numbers to NO remainders in the answer or carried (each digit must be a multiple of the divisor)

Remind children of the correct place value, that 96 is equal to 90 and 6, but in short division, pose:

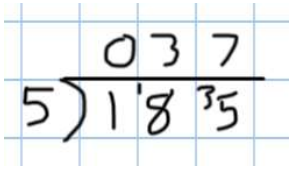
How many 3's in 9? = 3, and record it above the 9 tens.
How many 3's in 6? = 2, and record it above the 6 units.



Limited numbers to NO remainders in the final answer, but with remainders occurring within the calculation process.



Pupils need to move onto dividing m=numbers with up to 3-digitd by a single digit, however problems and calculations provided should not result in a final answer with a remainder at this stage.



When the answer for the first column is zero (1 ÷ 5, as an example), children could initially write a zero above to acknowledge its place, and must always 'carry' the number (1) over to the next digit as a remainder.

Real life contexts need to be used routinely to help pupils gain a full understanding, and the ability to recognise the place of division and how to apply it to problems.

Stage 4 – Year 5 Multiplication and Division

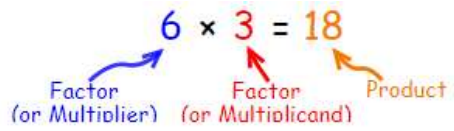
Key Learning:

- Choose an appropriate strategy to solve a calculation based upon the numbers involved (recall a known fact, calculate mentally, use a jotting, written method).
- Identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers.
- Know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers.
- Establish whether a number up to 100 is prime and recall prime numbers up to 19.
- Recognise and use square (^{2}) and cube (^{3}) numbers, and notation.
- Use partitioning to double or halve any number, including decimals to two decimal places.
- Multiply and divide numbers mentally drawing upon known facts.
- Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000.
- Solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes.
- Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers; Refine and use efficient written methods to multiply, for example HTU × U, TU × TU, U.t × U and TU.t h × TU.
- Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context; Refine and use efficient written methods to divide, for example HTU ÷U, U.t × U and HTU ÷ U.
- Use estimation/inverse to check answers to calculations; determine, in the context of a problem, an appropriate degree of accuracy.
- Solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign.
- Solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates.

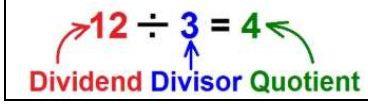
Key Vocabulary:

lots of, groups of
times, multiply, multiplication,
multiplied by
multiple of, product
once, twice, three times… ten times…
times as (big, long, wide… and so on)
repeated addition
array column
double,
row,

Multiplication



Division



halve
share, share equally
one each, two each, three each…
group in pairs, threes… tens
equal groups of
divide, division, divided by, divided into
remainder
factor, quotient, divisible by
inverse

Mental calculations:

Multiply and divide numbers mentally drawing upon known facts
Choose an appropriate strategy to solve a calculation based upon the numbers involved (recall a known fact, calculate mentally, use a jotting, written method).

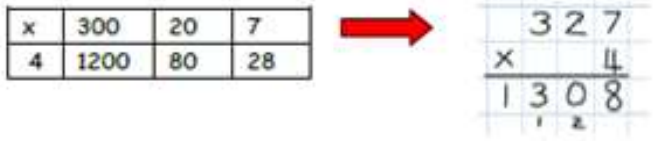
On the boil – Autumn	On the boil – Spring	On the boil – Summer
Use key vocabulary – sum, product, difference – to practice finding the sum, product difference of two single digit numbers. Rapid recall of all multiplication facts up to 12 x 12.	Use key vocabulary – sum, product, difference, square, cube to practice finding the sum, product, difference, square and cube of given numbers. Rapid recall of known factors within children's' multiplication facts.	Use key vocabulary – sum, product, difference, square, cube to practice finding the sum, product, difference, square and cube of given numbers. Identify prime numbers within a given range and square numbers up to 144.

Multiplication – Multiply up to 4-digits by 1 or 2 digits.

Column Multiplication

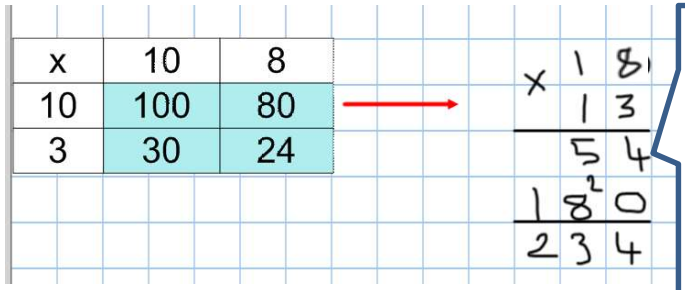
Introduce by comparing a grid method calculation to a short mulitplication method, to see how the steps are related, but notice how there are less steps involved in the column method.
Chidlren need to be taugthj to approximate first, e.g. for 72 x 38, they will use rounding: 72 x 38 is approximately 70 x 40 = 2800, and use this approximation to check the reasonableness of their answer against.

Short muliplication for multiplying by a single digit



Pupils could be asked to work out a given calculation using the grid method, and then compare it to 'your' column method. What are the similarities and differences? Unpick the steps and show how it reduces the steps.

Introduce long multiplication for multiplying by 2 digits



18 x 3 on the 1st row

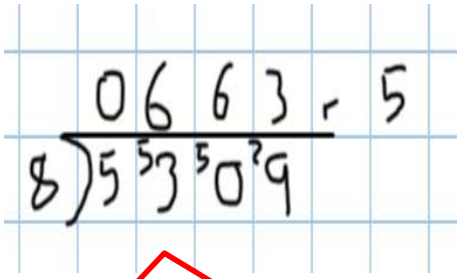
(8x3 = 24, carrying the 2 for twenty, then '1' x 3)

18 x 10 on the 2nd row

Put a zero in the units first, then say 8 x 1 and 1 x 1

Division – Divide up to 4 digits by a single digit, including those with reminders.

Short division using the bus-stop method, including remainders in the final answer.



Now that pupils are introduced to examples that give rise to remainder answers, division needs to have a real life problem solving context, where pupils consider the meaning of the remainder and how to express it. i.e. as a fraction, decimal, or as a rounded number or value depending upon the context of the problem.

The answer to 5309 ÷ 8 could be expressed as 663 and five eighths, 663 r 5, as a decimal or rounded as appropriate to the problem involved.

Stage 4 – Year 6 Multiplication and Division

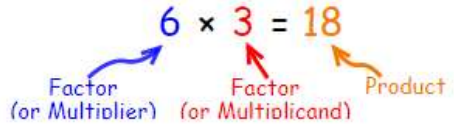
Key Learning:

- Choose an appropriate strategy to solve a calculation based upon the numbers involved (recall a known fact, calculate mentally, use a jotting, written method).
- Identify common factors, common multiples and prime numbers.
- Use partitioning to double or halve any number.
- Perform mental calculations, including with mixed operations and large numbers.
- Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication.
- Multiply one-digit numbers with up to two decimal places by whole numbers.
- Divide numbers up to 4 digits by a two-digit whole number using the formal written methods of short or long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context.
- Use written division methods in cases where the answer has up to two decimal places.
- Use estimation and inverse to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy.
  Use knowledge of the order of operations to carry out calculations.
- Solve problems involving all four operations, including those with missing numbers.

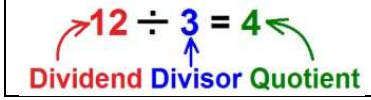
Key Vocabulary:

lots of, groups of
times, multiply, multiplication,
multiplied by
multiple of, product
once, twice, three times… ten times…
times as (big, long, wide… and so on)
repeated addition
array column
double,
row,

Multiplication



Division



halve
share, share equally
one each, two each, three each…
group in pairs, threes… tens
equal groups of
divide, division, divided by, divided into
remainder
factor, quotient, divisible by
inverse

Mental calculations:

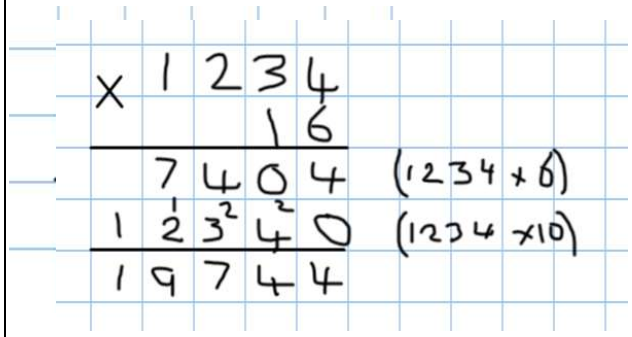
Choose an appropriate strategy to solve a calculation based upon the numbers involved (recall a known fact, calculate mentally, use a jotting, written method)
Perform mental calculations, including with mixed operations and large numbers

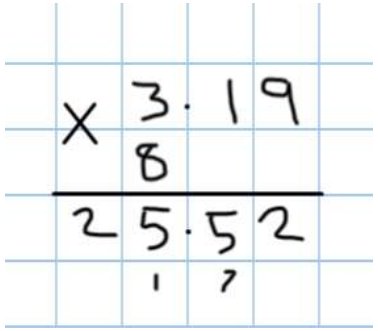
Use key vocabulary – sum, product, difference, square, cube to practice finding the sum, product, difference, square and cube of given numbers.

On the boil – Autumn	**On the boil – Spring**	**On the boil – Summer**
- Rapid recall of all multiplication facts up to 12 x 12 and their relevant division facts	– Rapid recall of known factors within children's' multiplication facts. - Deriving common factors for appropriate numbers	

Multiplication – Short and long multiplication, with more complex numbers and multiply decimals with up to 2d.p by a single digit.

Long mutliplication for more complex numbers





Children will be able to:

Use rounding and place value to make approximations before calculating and use these to check answers against.
Use short multiplication to multiply numbers with more than 4-digits by a single digit; to multiply money and measures, and to multiply decimals with up to 2.d.p. by a single digit.
Use long multiplication to multiply numbers with at least 4 digits by a 2-digit number.

This works well for multiplying money (£.p) and other measures.

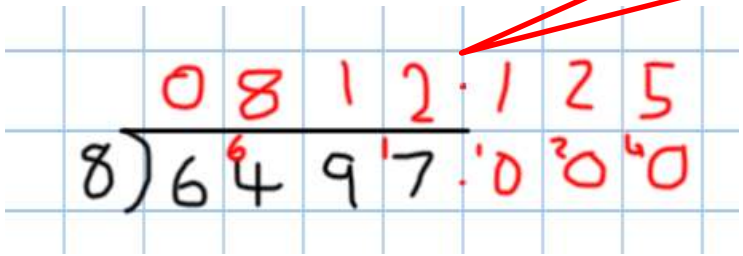
Division – Divide at least 4 digits by both single- digit and 2-digit numbers.

Short division with remainders: Pupils should continue to use this method, but with numbers to at least 4 digits, and understand how to express remainders as a fraction, decimal, whole number remainders or rounded numbers. Real life problems solving contexts need to be the starting point, where pupils have to consider the most appropriate way to express the remainder.

Short division, for dividing by a single digit:

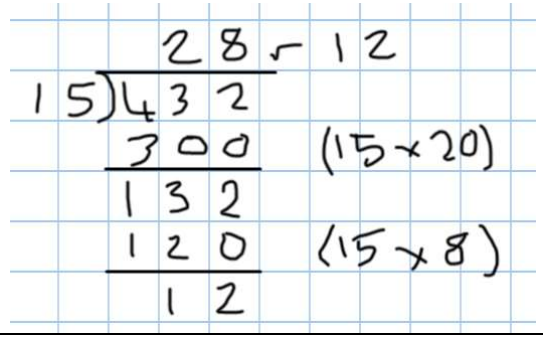
e.g. 4697 ÷ 8 =

Line up the decimal points in the question and the answer.



Calculating a decimal remainder: In this example, rather than expressing the remainder as r. 1 a decimal point is added after the units because there is still a remainder, and the one remainder is carried onto zeros after the decimal point (to show there was no decimal value in the original number). Keep dividing to an appropriate degree of accuracy for the problem being solved.

Introduce long division by chunking for dividing by 2 digits



Where remainders occur, pupils should express them as fractions, decimals or use rounding, depending upon the problem.

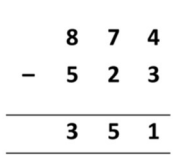
The following page is produced by the DFE and outline possible methods for the 4 operations. Our policy follows these guidelines and teaches pupils the steps leading up to these efficient methods.

Addition and subtraction

789 + 642 becomes

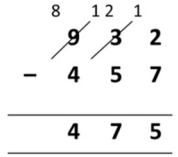
Answer: 1431

874 – 523 becomes



Answer: 351

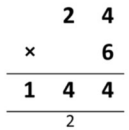
932 – 457 becomes



Answer: 475

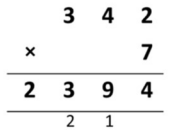
Short multiplication

24 × 6 becomes



Answer: 144

342 × 7 becomes



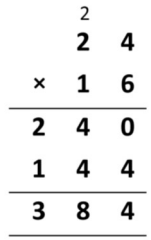
Answer: 2394

2741 × 6 becomes

Answer: 16 446

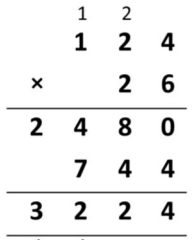
Long Multiplication

24 × 16 becomes



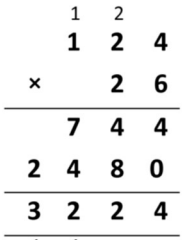
Answer: 384

124 × 26 becomes



Answer: 3224

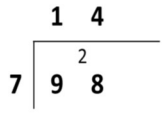
124 × 26 becomes



Answer: 3224

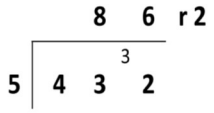
Short Division

98 ÷ 7 becomes



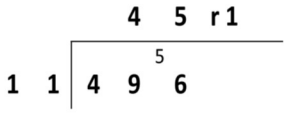
Answer: 14

432 ÷ 5 becomes



Answer: 86 remainder 2

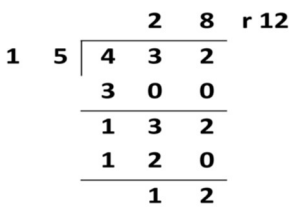
496 ÷ 11 becomes



Answer: $45\frac{1}{11}$

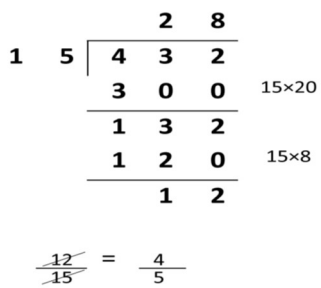
Long Division

432 ÷ 15 becomes



Answer: 28 remainder 12

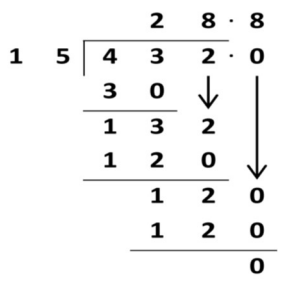
432 ÷ 15 becomes



$\frac{\cancel{12}}{\cancel{15}} = \frac{4}{5}$

Answer: $28\frac{4}{5}$

432 ÷ 15 becomes



Answer: 28·8